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When $p=r/s$, $2m/(p^2-2)=2ms^2/(r^2-2s^2)$.....(3).

When $p=2(r+s)/(r+2s)$, $2m/(p^2-2)=m(r+2s)^2(r^2-2s^2)$.....(4).

Since (3) is integral, (4) is also.

Also solved by the *PROPOSER*.

GEOMETRY.

200. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Find the locus of eight points of contact of the four common tangents of two concentric coaxial ellipses.

Solution by G. B. M. ZERR, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $x^2/a^2+y^2/b^2=1$, and $x^2/c^2+y^2/d^2=1$, be the equations to the ellipses, $c>a$, $b>d$.

Since $xx_1/a^2+yy_1/b^2=1$, and $xx_2/c^2+yy_2/d^2=1$, are two equations for the same line, we get $x_2=c^2x_1/a^2$, $y_2=d^2y_1/b^2$.

$\therefore x_1^2/a^2+y_1^2/b^2=1$, and $c^2x_1^2/a^4+d^2y_1^2/b^4=1$, give

$$x_1=\frac{a^2\sqrt{[b^2-d^2]}}{\sqrt{[b^2c^2-a^2d^2]}}=a^2m \text{ (suppose)},\ y_1=\frac{b^2\sqrt{[c^2-a^2]}}{\sqrt{[b^2c^2-a^2d^2]}}=b^2n,$$

$$x_2=c^2m,\ y_2=d^2n.$$

Hence $mx\pm ny\pm1=0$ represents the four common tangents. While $(c^2m,\ d^2n)$; $(a^2m,\ b^2n)$; $(-a^2m,\ b^2n)$; $(-c^2m,\ d^2n)$; $(-c^2m,\ -d^2n)$; $(a^2m,\ -b^2n)$; $(c^2m,\ -d^2n)$ are the eight points of contact. These eight points are situated on an ellipse, which is the locus required.

Let $x^2/A^2+y^2/B^2=1$ be this ellipse. Then $c^2m/A^2+d^4n^2/B^2=1$, also $a^4m^2/A^2+b^4n^2/B^2=1$.

$$\therefore A^2=\frac{(b^4c^4-a^4d^4)m^2}{b^4-d^4}=\frac{b^2c^2+a^2d^2}{b^2+d^2},\ B^2=\frac{(b^4c^4-a^4d^4)n^2}{c^4-a^4}=\frac{b^2c^2+a^2d^2}{a^2+c^2}.$$

$\therefore(b^2+d^2)x^2+(a^2+c^2)y^2=b^2c^2+a^2d^2$ is the locus.

201. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud. England.

Two plane sections of a right circular cone have their major axes AA', aa' coplanar, and Aa on one generator equal to $A'a'$ on the other. The projections of the sections on any plane perpendicular to the axis are confocal.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics. The Temple College, Philadelphia, Pa.

Let $FG=ED=r$, $FD=2A$, $GE=2a$, $\angle EGK=\varphi$, $\angle DFH=\theta$, $\angle GLF=x$, $\angle CGK=\angle CFH=\delta$, $CN=c$, $CM=d$.

Then $GF : LF=\sin x : \sin LGF$, or $r : LF=\sin x : \sin(\delta-\varphi)$.

Also $ED : LD=\sin x : \sin LED$, or $r : 2A+LF=\sin x : \sin(\delta+\varphi)$.

$\therefore LF\sin x=r\sin(\delta-\varphi)=r\sin(\delta+\varphi)-2A\sin x$.

$\therefore A\sin x=r\cos\delta\sin\varphi$.

Similarly, $a\sin x=r\cos\delta\sin\theta$. $\therefore A\sin\theta=a\sin\varphi$.

$GN+EN=2a=2c\cos\delta\sin\delta\cos\varphi/(\cos^2\varphi-\cos^2\delta)$.

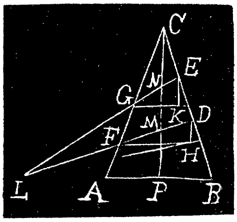
$\therefore c=a(\cos^2\varphi-\cos^2\delta)/\cos\delta\sin\delta\cos\varphi$,

$d=A(\cos^2\theta-\cos^2\delta)/\cos\delta\sin\delta\cos\theta$.

The axes of the projections on any plane perpendicular to CP are $A\cos\theta$ and B, and $a\cos\varphi$ and b, respectively.

$$\left.\begin{array}{l} x^2\cos^2\varphi(\tan^2\delta-\tan^2\varphi)+y^2\tan^2\delta+2cx\sin\varphi=c^2 \\ x^2\cos^2\theta(\tan^2\delta-\tan^2\theta)+y^2\tan^2\delta+2dx\sin\theta=d^2 \end{array}\right\}\text{ are the equations to the ellipses.}$$

$$\therefore A^2=\frac{d^2\cos^2\theta\sin^2\delta\cos^2\delta}{(\cos^2\theta-\cos^2\delta)^2},\quad B^2=\frac{d^2\cos^2\theta\cos^2\delta}{\cos^2\theta-\cos^2\delta}.$$



$$a^2=\frac{c^2\cos^2\varphi\sin^2\delta\cos^2\delta}{(\cos^2\varphi-\cos^2\delta)^2}=b^2=\frac{c^2\cos^2\varphi\cos^2\delta}{\cos^2\varphi-\cos^2\delta}.$$

Substituting values of d and c,

$$A^2\cos^2\theta-B^2=\frac{d^2\cos^2\theta\sin^2\theta\cos^4\delta}{(\cos^2\theta-\cos^2\delta)^2}=A^2\sin^2\theta\cot^2\delta.$$

$$a^2\cos^2\varphi-b^2=\frac{c^2\cos^2\varphi\sin^2\varphi\cos^4\delta}{(\cos^2\varphi-\cos^2\delta)^2}=a^2\sin^2\varphi\cot^2\delta=A^2\sin^2\theta\cot^2\delta.$$

$\therefore A^2\cos^2\theta-B^2=a^2\cos^2\varphi-b^2$, and the sections are confocal.

202. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

The equations $\sqrt{[l\alpha]}+\sqrt{[m\beta]}+\sqrt{[n\gamma]}=0$ and $l\beta\gamma+m\alpha\gamma+n\beta\alpha=0$ represent ellipses. If a, b, c are the sides of the triangle of reference, transform to Cartesian coördinates and find area of each ellipse.

Solution by the PROPOSER.

$\sqrt{[l\alpha]}+\sqrt{[m\beta]}+\sqrt{[n\gamma]}=0$, or

$$l^2\alpha^2+m^2\beta^2+n^2\gamma^2-2mn\beta\gamma-2nl\alpha\gamma-2ln\alpha\beta=0\,.....(1),$$

$$l\beta\gamma+m\gamma\alpha+n\alpha\beta=0\,.....(2).$$